The compressibility effects will be noticeable in the far flow field. To calculate these effects, recourse has to be made directly to the formula for ϕ in Eq. (3). For a large x and r value, the flight-path diagram will be the same as that shown in Fig. 1, but ξ_1 and ξ_2 will now be determined by the crossings of the curved trace of the retrograde Mach cone with the symmetric flight-path curves. In the far field, it will take a time $t = (1/c)\ (x^2 + r^2)^{1/2}$ before the first pressure pulse is felt in a given point (x,r), and the pressure variation in this point will thereafter be given by the t-derivative of Eq. (3) with ξ_1 and ξ_2 determined as indicated above.

For the cone, the expression for $\phi_t(x,r,t)$ in the far field is very complicated and will not be given here. However, one gets a simple formula by letting $r \to 0$, which is

$$(c_p)_{r=0} = k^2 \log \frac{(1 - M^2)x^2}{x^2 - U^2t^2} \text{ for } \left(\frac{x}{c}\right)^2 \leqq t^2 < \left(\frac{x}{U}\right)^2 \quad (10a)$$

$$(c_p)_{r=0} = 0 \text{ for } t^2 < \left(\frac{x}{c}\right)^2 \quad (10b)$$

Eqs. (10a) and (10b) give the time history of c_p for a fixed value of x.

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Characteristics of Lateral Range during Constant-Altitude Glide

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This note presents some results of calculations of lateral range during a constant-altitude glide and discusses their characteristics. The constant-altitude glide is maintained by continuously rolling the vehicle about its velocity vector at a fixed angle of attack. The applicable equations of motion are

$$(2W/C_DA\rho)(dV/dt) = -gV^2 \quad (1)$$

$$V^2(L/D)\cos\varphi = (2W/C_DA\rho)[1 - (V/V)_s^2] \quad (2)$$

$$\frac{d\psi}{dt} = \frac{L}{D}\frac{Vg\sin\varphi}{(2W/C_DA\rho)} - \frac{Vg\cos\psi\tan\lambda}{V_s^2} \quad (3)$$

$$d\lambda/dt = Vg\sin\psi/V_s^2 \quad (4)$$

The symbol φ is the roll angle measured from the local vertical, ψ is the turn angle measured from the vehicle's original heading, λ is the ratio of the lateral range (measured from the original great circle) to the radius of the earth, and V_s is the orbital speed. Other symbols have their usual meaning. The first term on the right-hand side of Eq. (3) is proportional to the lateral aerodynamic force, whereas the second term is proportional to the so-called lateral centrifugal force, which accounts for the effect of the earth's curvature in the lateral direction. Equation (3) can be deduced from Eq. (60) of Ref. 5.

These equations have been solved on a digital computer by Skulsky,[1] and some of the results are shown in Fig. 1 for a

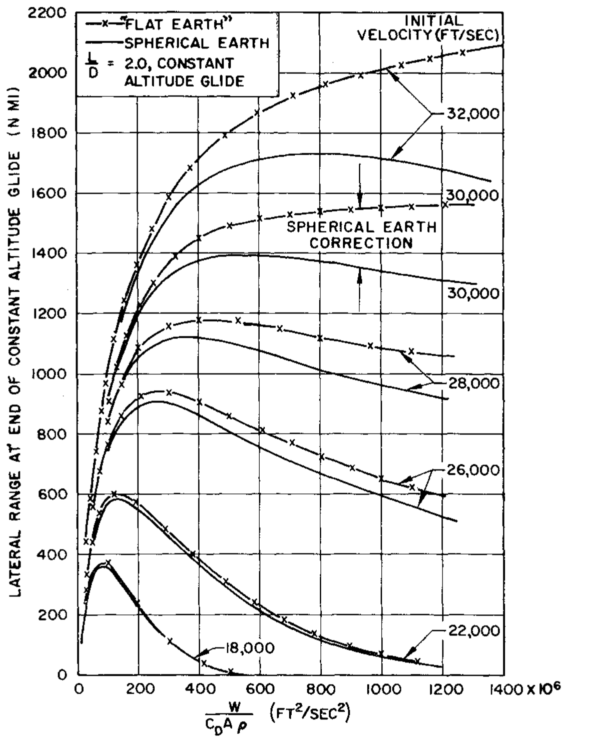

Fig. 1 Lateral range for various initial velocities and $W/C_DA\rho$

range of $W/C_DA\rho$ of practical interest. The lateral range presented is that obtained from the beginning of the constant-altitude glide at initial velocities indicated and an associated initial roll angle to the end of the glide, where the roll angle is zero. The results are for $L/D = 2$. Both the "flat-earth"† and spherical-earth calculations are presented.

It is well known that the lateral range during an equilibrium glide, keeping the vehicle at a constant roll angle and assuming a flat earth, is independent of W/C_DA (see Ref. 2). This independence, however, does not hold for constant-altitude glides, as shown in Fig. 1. During a constant-altitude glide, the required roll angle at any instant is a function of $W/C_DA\rho$ and L/D. This leads to the $W/C_DA\rho$ dependence of the turn angle ψ and hence the lateral range. This can be seen readily by the substitution of Eqs. (1) and (2) into (3).

Figure 1 reveals that the lateral range, as a function of $W/C_DA\rho$, exhibits maxima in all of the spherical-earth calculations and in most of the "flat-earth" calculations. Since maxima occur in the "flat-earth" cases, it is concluded that these maxima result fundamentally from the lateral aerodynamic-force term and can be understood by examining the lateral aerodynamic force only. For a given initial velocity, the turn angle ψ decreases as $W/C_DA\rho$ increases, as indicated by Eqs. (2) and (3). On the other hand, the total flight time from the beginning to the end of the constant-altitude glide increases as $W/C_DA\rho$ increases; this is true at least for the lower end of $W/C_DA\rho$. The velocity reduction during this time decreases as $W/C_DA\rho$ increases. Although the lateral range, as given by Eq. (4), is the time integral of $V\sin\psi$ (g and V_s essentially are constant), a maximum occurs as $W/C_DA\rho$ increases because of less velocity reduction, higher total time, and lower $\sin\psi$.

The data of Fig. 1 also show that the magnitude of lateral range is not always a satisfactory criterion for determining the validity of neglecting the spherical-earth correction. It is observed from Fig. 1 that for low values of $W/C_DA\rho$ the lateral range obtained by assuming a "flat earth" agrees fairly well with the spherical earth results, regardless of the magnitude of the lateral range and the initial velocity. The

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† In the flat-earth calculations, only the lateral centrifugal force was neglected.

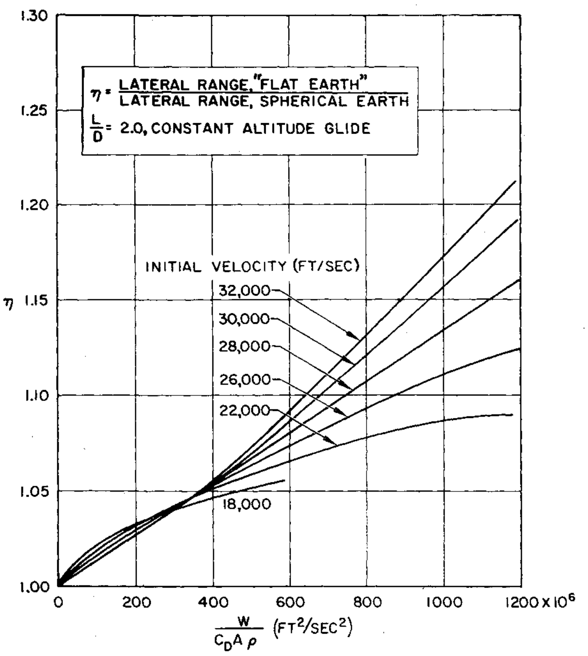

Fig. 2 Spherical earth correction on lateral range

"flat-earth" results also agree well with the spherical-earth results at higher $W/C_DA\rho$ if the initial velocity is low. At higher initial velocities and higher $W/C_DA\rho$, the spherical-earth correction becomes significant and causes a maximum lateral range to exist at a given initial velocity.

The question of what is the criterion for determining the significance of the lateral centrifugal force on lateral range has been discussed in Refs. 3 and 4 for the case of equilibrium glide. For an equilibrium glide, London[4] advanced an argument that seems to apply also in the case of the constant-altitude glide. He indicated that the proper criterion for determining the validity of neglecting the lateral centrifugal force is the magnitude of the time integral of the ratio of this force to the lateral aerodynamic force, whereas the magnitude of the lateral range is not in itself a sufficient criterion. This seems to explain the characteristics of this correction noted previously. For a given initial velocity, the lateral aerodynamic force is large when $W/C_DA\rho$ is low, as can be seen from Eqs. (2) and (3). As a result, the lateral centrifugal force is never significant compared to the lateral aerodynamic force when $W/C_DA\rho$ is low and the correction is negligible. Equation (3) also shows that the lateral centrifugal force is approximately proportional to $\tan\lambda$, and consequently this force does not change appreciably for equal lateral range. For equal lateral range, therefore, the lateral-centrifugal-force correction should become larger at higher $W/C_DA\rho$ because the lateral aerodynamic force is lower, whereas the lateral centrifugal force stays relatively constant. This feature is shown clearly for the cases of high initial velocities of Fig. 1. A summary of the lateral-centrifugal-force correction is shown in Fig. 2.

The discussion presented in this note is for $L/D = 2$. Similar characteristics, although not shown here, have been observed for $L/D = 0.5$, 1.0, 1.5, and 2.5. The foregoing findings can be summarized as follows:

1) The lateral range obtained during a constant-altitude glide over a spherical earth varies with $W/C_DA\rho$ and exhibits a maximum for a given initial velocity.

2) The spherical-earth correction is negligible at low $W/C_DA\rho$, regardless of the magnitude of lateral range and initial velocity, and becomes significant at high $W/C_DA\rho$ and high initial velocities.

3) The criterion for determining the significance of the lateral centrifugal force on lateral range seems to be the relative magnitude of the time integral of this force to the lateral aerodynamic force. This agrees with London's observation in the case of equilibrium glide.

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Forces due to the Magnetic Field of the Electrical Conductivity Meter

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A CONDUCTOR moving through an applied magnetic field causes induced currents. The magnitude of the induced magnetic field is a measure of the product of the electrical conductivity σ and velocity u of the conductor. An instrument to measure σu was flown aboard a re-entry vehicle.[1, 2] The interaction of the moving conductor with the applied magnetic field also produces a force on the magnet; this force is a desirable effect for magnetoaerodynamic attitude control. However, for plasma-sheath measurement, the force is undesirable since it eventually could perturb the trajectory of the re-entry vehicle. The theory predicts a rather small force, but it is of interest to measure the force directly on the σu transducer for conditions simulating flight.

The x component of the force on the magnet is

$$F_x = \int_y \sigma(y)U(y) \int_x\int_z [B_z^2(x, y,z) + B_y^2(x,y,z)]dxdydz \quad (1)$$

A right-handed coordinate system has been chosen with the x axis parallel to the flow and the y axis normal to the surface. The electric field is negligible (see Ref. 2) and, hence, has been neglected in the derivation of Eq. (1). The magnetic field B is proportional to the current in the primary coils i. Equation (1) shows that F_x is proportional to σ, to u, and to i^2.

The components of the magnetic field of the flight transducer were measured with an aluminum plate covering the transducer. The aluminum plate duplicates the skin of the re-entry vehicle. To simulate the plasma flow, a spinning graphite disk was employed. Using the measured B_y and B_z and values of σ and u appropriate for the graphite disk, Eq. (1) was evaluated numerically. The results are shown in Fig. 1.

The force was determined experimentally using apparatus shown schematically in Fig. 2. Thin metal strips formed the hinges of a parallelogram suspension. Displacements, the order of 0.001 in., were determined by a differential transformer. The apparatus was adapted from a previous ex-

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